15^{th} Graduate Colloquium in Mathematics

Universität Bern

Abstracts

Annina Iseli (Universität Bern) — *Marstrand's theorem on Riemannian manifolds.*
Marstrand's theorem states that for a given compact set A in $\mathbb{R}^2$ of dimension $d \leq 1$, its image $P(A)$ under the orthogonal projection P onto some line L through the origin also is a set of dimension d for almost every such line L. In this talk I will recall a proof of Marstrand's theorem and give an overview of more general and stronger versions. Finally, I will discuss the possibility of achieving analogous results in other metric spaces, in particular on Riemannian manifolds.

Ali Hyder (Universität Basel) — *Structure of conformal metrics on $\mathbb{R}^n$ with constant Q-curvature and finite volume.*
We are interested in the following problem

$$(-\Delta)^{\frac{n}{2}}u = (n-1)!e^{nu} \quad \text{in } \mathbb{R}^n, \quad V = \int_{\mathbb{R}^n} e^{nu}dx < \infty,$$

which is a higher dimensional analog of the prescribed Gaussian curvature equation. Geometrically, if u is a smooth solution to the above equation then the conformal metric $g_u = e^{2u}|dx|^2$ has the constant Q-curvature equal to $(n-1)!$ and finite volume V.
The operator $(-\Delta)^{\frac{n}{2}}$ is a non-local operator for odd integer n. We will discuss the difference between the even-dimensional and odd-dimensional cases. Depending on the time we will present some existence and classification results.

Alberto Ravagnani (Université de Neuchâtel) — *Equidistant codes in network coding.*
Coding theory was invented to allow reliable communications over noisy channels. Applications include satellite communications, mobile phones, optical fibers transmissions, flash memories and compact discs. Within coding theory, network coding focuses on the situation where one source attempts to transmit several messages to multiple receivers through a noisy network of intermediate nodes. In this context, a subspace code is defined as a collection of vector spaces of fixed dimension over a finite field. We concentrate on families of subspaces whose pairwise intersections all have the same dimension, and classify the maximal ones for most choices of the parameters, showing that they have a very simple structure. Moreover, we propose a simple construction for them than can be easily manipulated by a computer.

Anthony Conway (Université de Genève) — *Colored tangles and signatures.*

In this talk, I will discuss signature invariants of links. More precisely, as links can always be obtained from tangles via the closure operation, I will relate these invariants to the Burau representation of the braid group.

Prof. Dr. Eva Bayer-Fluckiger (École polytechnique fédérale de Lausanne) — *The Euclidean division.*

If a and b are two integers, with $b \neq 0$, then there exist two integers q and r such that $a = bq + r$, and that $|r| < |b|$. This so-called Euclidean division property plays a fondamental role in the arithmetic of the usual integers. It is natural to try to generalise this to more general rings, for instance rings of integers of algebraic number fields. This idea leads to the notions of Euclidean number fields and Euclidean minima. Both are very classical topics of number theory. The aim of this talk is to survey old and new results concerning this subject, such as new Euclidean number fields and upper bounds for Euclidean minima. In particular, we will survey the history and recent developments concerning a classical conjecture of Minkowski.

Filip Misev (Universität Bern) — *Twisted bands in embedded surfaces.*

Let S be a surface with boundary in $\mathbb{R}^3$. Consider an unknotted curve in S whose neighbourhood in S is a twisted band. The set of different such bands with some fixed amount of twisting contains information about the surface S and its boundary knot. For example, it can be infinite, finite, or even empty. For positive tree-like Hopf plumbings (a certain class of surfaces constructed from finite trees), this classification into finite/infinite band sets turns out to be closely related to the classification of finite/infinite Coxeter groups.

Claudiu Vaculescu (École polytechnique fédérale de Lausanne) — *Distinct and repeated distance problems in the plane.*

We start the talk by presenting two central problems in Discrete Geometry, posed by Paul Erdos in 1946: the unit and the distinct distance problem, together with the well-known Szemeredi-Trotter incidence theorem. Having these as starting point, we sketch the proof of the following original result: For any set P of m points, and for any set L of n lines, both in the plane, the number of distinct distances between points of P and lines of L is at least $cm^{1/5}n^{3/5}$, for some constant c. This is joint work with Micha Sharir, Shakhar Smorodinsky, and Frank de Zeeuw.

Dimitri Wyss (École polytechnique fédérale de Lausanne) — *Prime numbers in complex geometry.*

In my talk I will explain how the number of point of an algebraic variety over a finite field can give you information about the cohomology of the same variety over the complex numbers. This allows us to apply a Fourier transform technique which, at first sight, has nothing to do with geometry, but is nevertheless very effecitve in computing cohomology.

Elia Saini (Université de Fribourg) — *Uniform Hyperplane Arrangements.*

We present some results on the interplay between combinatorics and topology in the study of the complement manifold of complex hyperplane arrangements. In particular, we show that the complements of complex central hyperplane arrangements with the same associated uniform ma- troid are diffeomorphic.

Johannes Josi (Université de Genève) — *Real algebraic curves of degree 6 in the projective plane.*

The topology of real algebraic curves in the plane, while already studied by Klein and Hilbert, remains quite mysterious today. I will focus on the case of sextics (= curves of degree six), which is quite special because they correspond to a class of complex surfaces, called K3, whose deformations are well-understood thanks to works of Piatetski-Shapiro, Shafarevich and others in the seventies. I will explain how this allowed Nikulin (1979) to classify smooth sextics and how it leads to a correspondence between sextics with certain singularities and faces of hyperbolic polytopes.